Signals and Systems
What is a Signal?

• A signal is a pattern of variation of some form
• Signals are variables that carry information
Examples

- Electrical signals
  - Voltages and currents in a circuit
- Acoustic signals
  - Acoustic pressure (sound) over time
- Mechanical signals
  - Velocity of a car over time
- Video signals
  - Intensity level of a pixel (camera, video) over time
Signal

- The Speech Signal

- The ECG Signal
Signal
– The image
Signal

– The image
Signal

• It is the variation pattern that conveys the information, in a signal

• Signal may exist in many forms like acoustic, image, video, electrical, heat & light signal
How is a Signal Represented?

- Mathematically, signals are represented as a function of one or more **independent variables**.
- For instance a black & white video signal intensity is dependent on $x$, $y$ coordinates and time $t$ $f(x,y,t)$
- On this course, we shall be exclusively concerned with signals that are a function of a single variable: $f(t)$
What is a Signal?

• (DEF) Signal : A signal is formally defined as a function of one or more variables, which conveys information on the nature of physical phenomenon.
Mathematical Background

• To understand this course it is needed to know
• Complex numbers
• Derivatives
• Integral
• Linear algebra
• Differential equations
Complex numbers

A complex number $z$ represents any point $(x, y)$ in a two-dimensional plane by $z = x + jy$, where $x = \Re[z]$ (real part of $z$) is the coordinate in the $x$ axis and $y = \Im[z]$ (imaginary part of $z$) is the coordinate in the $y$ axis. The symbol $j = \sqrt{-1}$ just indicates that $z$ needs to have two components to represent a point in the two-dimensional plane. Interestingly, a vector $\vec{z}$ that emanates from the origin of the complex plane $(0, 0)$ to the point $(x, y)$ with a length

$$|\vec{z}| = \sqrt{x^2 + y^2} = |z|$$

and an angle

$$\theta = \angle \vec{z} = \angle z$$
Polar form

also represents the point \((x, y)\) in the plane and has the same attributes as the complex number \(z\). The couple \((x, y)\) is therefore equally representable by the vector \(\vec{z}\) or by a complex number \(z\) that can be written in a rectangular or in a polar form,

\[
z = x + jy = |z|e^{j\theta}
\]
Euler Identity

\[ e^{j\theta} = \cos(\theta) + j \sin(\theta) \]
Cos and Sin

The relation between the complex exponentials and the sinusoidal functions is of great importance in signals and systems analysis. Using Euler’s identity the cosine can be expressed as

\[
\cos(\theta) = \mathcal{R}e\left[e^{j\theta}\right] = \frac{e^{j\theta} + e^{-j\theta}}{2}
\]

while the sine is given by

\[
\sin(\theta) = \mathcal{I}m\left[e^{j\theta}\right] = \frac{e^{j\theta} - e^{-j\theta}}{2j}
\]
These relations can be used to define the hyperbolic sinusoids as

\[ \cos(j\alpha) = \frac{e^{-\alpha} + e^{\alpha}}{2} = \cosh(\alpha) \]

\[ j\sin(j\alpha) = \frac{e^{-\alpha} - e^{\alpha}}{2} = -\sinh(\alpha) \]
Euler’s identity can also be used to find different trigonometric identities. For instance,

\[
\cos^2(\theta) = \left[ \frac{e^{j\theta} + e^{-j\theta}}{2} \right]^2 = \frac{1}{4} \left[ 2 + e^{j2\theta} + e^{-j2\theta} \right] = \frac{1}{2} + \frac{1}{2} \cos(2\theta)
\]

\[
\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{1}{2} - \frac{1}{2} \cos(2\theta)
\]

\[
\sin(\theta) \cos(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} = \frac{e^{j\theta} + e^{-j\theta}}{2} = \frac{e^{j2\theta} - e^{-j2\theta}}{4j} = \frac{1}{2} \sin(2\theta)
\]
Elementary Signals

- Exponential signals \( x(t) = Be^{at} \)
- Sinusoidal signals \( x(t) = A \cos(\omega t + \phi) \)
- Exponentially damped sinusoidal signals \( x(t) = Ae^{at} \cos(\omega t + \phi) \)
A sinusoid $x(t)$ is a periodic signal represented by

$$x(t) = A \cos(\Omega_0 t + \psi) \quad -\infty < t < \infty$$

where $A$ is the amplitude, $\Omega_0 = 2\pi f_0$ is the frequency in rad/sec, and $\psi$ is the phase in radians. The signal $x(t)$ is defined for all values of $t$, and it repeats periodically with a period $T_0 = 1/f_0$ (sec), so that $f_0$ is the frequency in cycles/sec or in Hertz (Hz) (in honor of H. R. Hertz). Given that the units of $\Omega_0$ is rad/sec, then $\Omega_0 t$ has as units (rad/sec) $\times$ (sec) = (rad), which coincides with the units of the phase $\psi$, and permits the computation of the cosine. If $\psi = 0$, then $x(t)$ is a cosine, and if $\psi = -\pi/2$, then $x(t)$ is a sine.
Elementary Signals

- Step function

\[ x(t) = u(t) \]
(a) Rectangular pulse $x(t)$ of amplitude $A$ and duration of 1 s, symmetric about the origin. (b) Representation of $x(t)$ as the difference of two step functions of amplitude $A$, with one step function shifted to the left by $\frac{1}{2}$ and the other shifted to the right by $\frac{1}{2}$; the two shifted signals are denoted by $x_1(t)$ and $x_2(t)$, respectively. Note that $x(t) = x_2(t) - x_1(t)$. 
Elementary Signals

• **Impulse function** \( x(t) = \delta(t) \)

(a) Evolution of a rectangular pulse of unit area into an impulse of unit strength (i.e., unit impulse). (b) Graphical symbol for unit impulse. (c) Representation of an impulse of strength \( a \) that results from allowing the duration \( \Delta \) of a rectangular pulse of area \( a \) to approach zero.
Elementary Signals

- Ramp function

\[ x(t) = r(t) \]
Classification of Signals

- Continuous and discrete-time signals
- Even and odd signals
- Periodic signals, non-periodic signals
- Deterministic signals, random signals
- Causal and anticausal signals
- Right-handed and left-handed signals
- Finite and infinite length
Continuous-Time (Analog) Signals

- Most signals in the real world are continuous time.
- Eg voltage, velocity,
- Denote by $x(t)$, where the time interval may be bounded (finite) or infinite
Discrete-Time(Digital) Signals

- Some real world and many digital signals are discrete time, as they are sampled.
- E.g. pixels, daily stock price (anything that a digital computer processes).
- Denote by $x[n]$, where $n$ is an integer value that varies discretely.

![Graph](image-url)
Deterministic signals, random signals

- **Deterministic signals**
  - There is no uncertainty with respect to its value at any time. (ex) $\sin(3t)$

- **Random signals**
  - There is *uncertainty* before its actual occurrence.
Causal and anticausal Signals

• Causal signals: zero for all negative time
• Anticausal signals: zero for all positive time
• Noncausal: nonzero values in both positive and negative time
Right-handed and left-handed Signals

- Right-handed and left handed-signal: zero between a given variable and positive or negative infinity
Finite and infinite length

- **Finite-length signal**: nonzero over a finite interval $t_{\text{min}} < t < t_{\text{max}}$
- **Infinite-length signal**: nonzero over all real numbers
Basic Operations on Signals

- Operations performed on dependent signals
- Operations performed on the independent signals
Operations performed on dependent signals

- Amplitude scaling: \[ y(t) = cx(t) \]
- Addition: \[ y(t) = x_1(t) + x_2(t) \]
- Multiplication: \[ y(t) = x_1(t) \cdot x_2(t) \]
- Differentiation: \[ y(t) = \frac{d}{dx} x(t) \]
- Integration: \[ y(t) = \int_{-\infty}^{t} x(\tau) d\tau \]
Operations performed on the independent signals

- **Time scaling** \( y(t) = x(at) \)
  - \( a > 1 \) : compressed
  - \( 0 < a < 1 \) : expanded
Operations performed on the independent signals

- Reflection \( y(t) = x(-t) \)
Operations performed on the independent signals

- **Time shifting** $y(t) = x(t - t_0)$

- **Precedence Rule for time shifting & time scaling**

  $y(t) = x(at - b) = x(a(t - \frac{b}{a}))$
• Example.

• Draw the graphic of the signal $y(t) = x(2t + 3)$ by the precedence rule, if graphic of the signal $x(t)$ is given
The incorrect way of applying the precedence rule. (a) Signal $x(t)$. (b) Time-scaled signal $v(t) = x(2t)$. (c) Signal $y(t)$ obtained by shifting

$v(t) = x(2t)$ by 3 time units, which yields $y(t) = x(2(t + 3))$.

The proper order in which the operations of time scaling and time shifting (a) Rectangular pulse $x(t)$ of amplitude 1.0 and duration 2.0, symmetric about the origin. (b) Intermediate pulse $v(t)$, representing a time-shifted version of $x(t)$. (c) Desired signal $y(t)$, resulting from the compression of $v(t)$ by a factor of 2.
Transformations of the independent variables

Given an analog signal \( x(t) \) and \( \tau > 0 \) we have that with respect to \( x(t) \):

(a) \( x(t - \tau) \) is delayed or shifted right \( \tau \) seconds.
(b) \( x(t + \tau) \) is advanced or shifted left \( \tau \) seconds.
(c) \( x(-t) \) is reflected.
(d) \( x(-t - \tau) \) is reflected and shifted left \( \tau \) seconds, while \( x(-t + \tau) \) is reflected and shifted right \( \tau \) seconds.

![Graphs showing transformations](image_url)
Example

Consider an analog pulse

\[ x(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise} 
\end{cases} \]

Find mathematical expressions for \( x(t) \) delayed by 2, advanced by 2, and the reflected signal \( x(-t) \).
Consider an analog pulse

\[ x(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases} \]

Find mathematical expressions for \( x(t) \) delayed by 2, advanced by 2, and the reflected signal \( x(-t) \).

The delayed signal \( x(t - 2) \) can be found mathematically by replacing the variable \( t \) by \( t - 2 \) so that

\[ x(t - 2) = \begin{cases} 
1 & 0 \leq t - 2 \leq 1 \text{ or } 2 \leq t \leq 3 \\
0 & \text{otherwise}
\end{cases} \]

The value \( x(0) \) (which in \( x(t) \) occurs at \( t = 0 \)) in \( x(t - 2) \) now occurs when \( t = 2 \), so that the signal \( x(t) \) has been shifted to the right two units of time, and since the values are occurring later, the signal \( x(t - 2) \) is said to be “delayed” by 2 with respect to \( x(t) \).
Example

Consider an analog pulse

\[ x(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases} \]

Find mathematical expressions for \( x(t) \) delayed by 2, advanced by 2, and the reflected signal \( x(-t) \).

Likewise, we have that

\[ x(t + 2) = \begin{cases} 
1 & 0 \leq t + 2 \leq 1 \text{ or } -2 \leq t \leq -1 \\
0 & \text{otherwise}
\end{cases} \]

The signal \( x(t + 2) \) can be seen to be the advanced version of \( x(t) \), as it is this signal shifted to the left by two units of time. The value \( x(0) \) for \( x(t + 2) \) now occurs at \( t = -2 \), which is ahead of \( t = 0 \).
Example

Consider an analog pulse

\[ x(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases} \]

Find mathematical expressions for \( x(t) \) delayed by 2, advanced by 2, and the reflected signal \( x(-t) \).

Finally, the signal \( x(-t) \) is given by

\[ x(-t) = \begin{cases} 
1 & 0 \leq -t \leq 1 \text{ or } -1 \leq t \leq 0 \\
0 & \text{otherwise}
\end{cases} \]

This signal is a mirror image of the original: the value \( x(0) \) still occurs at the same time, but \( x(1) \) occurs when \( t = -1 \).
Even and odd signals

• Even whenever coincides with its reflection \( x(-t) \)
• Odd whenever \( x(t) \) coincides with \(-x(-t)\) that is the negative of its reflection.
• Even \( x(-t)=x(t) \)
• Odd \( x(-t)=-x(t) \)
Even and odd decomposition: Any signal $y(t)$ is representable as a sum of an even component $y_e(t)$ and an odd component $y_o(t)$:

\[ y(t) = y_e(t) + y_o(t) \]

\[ y_e(t) = 0.5 \left[ y(t) + y(-t) \right] \]

\[ y_o(t) = 0.5 \left[ y(t) - y(-t) \right] \]
Example

Consider the analog signal

\[ x(t) = \cos(2\pi t + \theta) \quad -\infty < t < \infty \]

Determine the value of \( \theta \) for which \( x(t) \) is even and odd. If \( \theta = \pi/4 \), is \( x(t) = \cos(2\pi t + \pi/4) \), \(-\infty < t < \infty\), even or odd?
The reflection of $x(t)$ is $x(-t) = \cos(-2\pi t + \theta)$. Then:

1. $x(t)$ is even if $x(t) = x(-t)$ or
   \[
   \cos(2\pi t + \theta) = \cos(-2\pi t + \theta) \\
   = \cos(2\pi t - \theta)
   \]
   or $\theta = -\theta$ or $\theta = 0, \pi$. Thus, $x_1(t) = \cos(2\pi t)$ as well as $x_2(t) = \cos(2\pi t + \pi) = -\cos(2\pi t)$ are even.

2. for $x(t)$ to be odd, we need that $x(t) = -x(-t)$ or
   \[
   \cos(2\pi t + \theta) = -\cos(-2\pi t + \theta) = \cos(-2\pi t + \theta \pm \pi) = \cos(2\pi t - \theta \mp \pi)
   \]
   which can be obtained with $\theta = -\theta \mp \pi$ or $\theta = \mp \pi/2$. Indeed, $\cos(2\pi t - \pi/2) = \sin(2\pi t)$ and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$ are both odd. Thus, $x_3(t) = \pm \sin(2\pi t)$ is odd.

When $\theta = \pi/4$, $x(t) = \cos(2\pi t + \pi/4)$ is neither even nor odd according to the above. \(\blacksquare\)
Example

Consider the signal

\[ x(t) = \begin{cases} 
2 \cos(4t) & t > 0 \\
0 & \text{otherwise}
\end{cases} \]

Find its even and odd decomposition. What would happen if \( x(0) = 2 \) instead of 0—that is, when we define the sinusoid at \( t = 0 \)? Explain.
The signal $x(t)$ is neither even nor odd given that its values for $t \leq 0$ are zero. For its even-odd decomposition, the even component is given by

$$x_e(t) = 0.5[x(t) + x(-t)]$$

$$= \begin{cases} 
\cos(4t) & t > 0 \\
\cos(4t) & t < 0 \\
0 & t = 0 
\end{cases}$$

and the odd component is given by

$$x_o(t) = 0.5[x(t) - x(-t)]$$

$$= \begin{cases} 
\cos(4t) & t > 0 \\
-\cos(4t) & t < 0 \\
0 & t = 0 
\end{cases}$$

which when added together become the given signal.
If $x(0) = 2$, we have

$$x_e(t) = 0.5[x(t) + x(-t)]$$

$$= \begin{cases} 
\cos(4t) & t > 0 \\
\cos(4t) & t < 0 \\
2 & t = 0 
\end{cases}$$

while the odd component is the same. The even component has a discontinuity at $t = 0$. 
Periodic signals

• a signal is periodic if it repeats itself after a fixed period $T$, i.e. $x(t) = x(t+T)$ for all $t$.
• For example, $\sin(t)$ signal is periodic.
Periodic signals, non-periodic signals

- **Periodic signals**
  - A function that satisfies the condition $x(t) = x(t + T)$ for all $t$
  - Fundamental frequency: $f = 1/T$
  - Angular frequency: $\omega = 2\pi / T$

- **Non-periodic signals**
Consider the analog sinusoid

\[ x(t) = A \cos(\Omega_0 t + \theta) \quad -\infty < t < \infty \]

Determine the period of this signal, and indicate for what frequency \( \Omega_0 \) the period of \( x(t) \) is not clearly defined.
The analog frequency is $\Omega_0 = 2\pi / T_0$ so $T_0 = 2\pi / \Omega_0$ is the period. Whenever $T_0 > 0$ (or $\Omega_0 > 0$) these sinusoidal signals are periodic. For instance, consider

$$x(t) = 2 \cos(2t - \pi/2) \quad -\infty < t < \infty$$

Its period is found by noticing that this signal has an analog frequency $\Omega_0 = 2 = 2\pi f_0$ (rad/sec), or a hertz frequency of $f_0 = 1/\pi = 1/T_0$, so that $T_0 = \pi$ is the period in seconds. That this is the period can be seen for an integer $N$,

$$x(t + NT_0) = 2 \cos(2(t + NT_0) - \pi/2) = 2 \cos(2t + 2\pi N - \pi/2)$$

$$= 2 \cos(2t - \pi/2) = x(t)$$

since adding $2\pi N$ (a multiple of $2\pi$) to the angle of the cosine gives the original angle. If $\Omega_0 = 0$—that is, dc frequency—the period cannot be defined because of the division by zero when finding $T_0 = 2\pi / \Omega_0$. 

\[ \square \]
Consider a periodic signal $x(t)$ of period $T_0$. Determine whether the following signals are periodic, and if so, find their corresponding periods:

(a) $y(t) = A + x(t)$.
(b) $z(t) = x(t) + v(t)$ where $v(t)$ is periodic of period $T_1 = NT_0$, where $N$ is a positive integer.
(c) $w(t) = x(t) + u(t)$ where $u(t)$ is periodic of period $T_1$, not necessarily a multiple of $T_0$. Determine under what conditions $w(t)$ could be periodic.
(a) Adding a constant to a periodic signal does not change the periodicity, so \( y(t) \) is periodic of period \( T_0 \)—that is, for an integer \( k \), \( y(t + kT_0) = A + x(t + kT_0) = A + x(t) \) since \( x(t) \) is periodic of period \( T_0 \).

(b) The period \( T_1 = NT_0 \) of \( v(t) \) is also a period of \( x(t) \), and so \( z(t) \) is periodic of period \( T_1 \) since for any integer \( k \),

\[
    z(t + kT_1) = x(t + kT_1) + v(t + kT_1) = x(t + kNT_0) + v(t) = x(t) + v(t)
\]

given that \( v(t + kT_1) = v(t) \), and that \( kN \) is an integer so that \( x(t + kNT_0) = x(t) \). The periodicity can be visualized by considering that in one period of \( v(t) \) we can place \( N \) periods of \( x(t) \).

(c) The condition for \( w(t) \) to be periodic is that the ratio of the periods of \( x(t) \) and of \( u(t) \) be

\[
    \frac{T_1}{T_0} = \frac{N}{M}
\]
Example

Let $x(t) = e^{j2t}$ and $y(t) = e^{j\pi t}$, and consider their sum $z(t) = x(t) + y(t)$, and their product $w(t) = x(t)y(t)$. Determine if $z(t)$ and $w(t)$ are periodic, and if so, find their periods. Is $p(t) = (1 + x(t))(1 + y(t))$ periodic?
According to Euler’s identity,

\[ x(t) = \cos(2t) + j \sin(2t) \]
\[ y(t) = \cos(\pi t) + j \sin(\pi t) \]

indicating \( x(t) \) is periodic of period \( T_0 = \pi \) (the frequency of \( x(t) \) is \( \Omega_0 = 2 = 2\pi / T_0 \)) and \( y(t) \) is periodic of period \( T_1 = 2 \) (the frequency of \( y(t) \) is \( \Omega_1 = \pi = 2\pi / T_1 \)).

For \( z(t) \) to be periodic requires that \( T_1 / T_0 \) be a rational number, which is not the case as \( T_1 / T_0 = 2 / \pi \). So \( z(t) \) is not periodic.

The product is \( w(t) = x(t)y(t) = e^{j(2+\pi)t} = \cos(\Omega_2 t) + j \sin(\Omega_2 t) \) where \( \Omega_2 = 2 + \pi = 2\pi / T_2 \) so that \( T_2 = 2\pi / (2 + \pi) \), so \( w(t) \) is periodic of period \( T_2 \).

The terms \( 1 + x(t) \) and \( 1 + y(t) \) are periodic of period \( T_0 = \pi \) and \( T_1 = 2 \), and from the case of the product above, one would hope this product be periodic. But since \( p(t) = 1 + x(t) + y(t) + x(t)y(t) \) and \( x(t) + y(t) \) is not periodic, then \( p(t) \) is not periodic.
Another possible classification of signals is based on their energy and power. The concepts of energy and power introduced in circuit theory can be extended to any signal. Recall that for a resistor of unit resistance its instantaneous power is given by

\[ p(t) = v(t)i(t) = i^2(t) = v^2(t) \]

where \( i(t) \) and \( v(t) \) are the current and voltage in the resistor. The energy in the resistor for an interval \([t_0, t_1]\), of duration \( T = t_1 - t_0 \), is the accumulation of instantaneous power over that time interval,

\[ E_T = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} i^2(t)dt = \int_{t_0}^{t_1} v^2(t)dt \]
The *power* in the interval $T = t_1 - t_0$ is the average energy

$$P_T = \frac{E_T}{T} = \frac{1}{T} \int_{t_0}^{t_1} i^2(t)dt = \frac{1}{T} \int_{t_0}^{t_1} v^2(t)dt$$

corresponding to the heat dissipated by the resistor (and for which you pay the electric company). The energy and power concepts can thus be easily generalized.
The energy and the power of an analog signal $x(t)$ are defined for either finite or infinite-support signals as:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

The signal $x(t)$ is then said to be finite energy, or square integrable, whenever

$$E_x < \infty$$

The signal is said to have finite power if

$$P_x < \infty$$
Find the energy and the power of the following:

(a) The periodic signal \( x(t) = \cos(\pi t/2 + \pi/4) \).
(b) The complex signal \( y(t) = (1 + j)e^{j\pi t/2} \), for \( 0 \leq t \leq 10 \) and zero otherwise.
(c) The pulse \( z(t) = 1 \), for \( 0 \leq t \leq 10 \) and zero otherwise.

Determine whether these signals are finite energy, finite power, or both.
The energy in these signals is computed as follows:

\[ E_x = \int_{-\infty}^{\infty} \cos^2(\pi t/2 + \pi/4) dt \to \infty \]

\[ E_y = \int_{0}^{10} |(1 + j)e^{j\pi t/2}|^2 dt = 2 \int_{0}^{10} dt = 20 \]

\[ E_z = \int_{0}^{10} dt = 10 \]

where we used \( |(1 + j)e^{j\pi t/2}|^2 = |1 + j|^2 |e^{j\pi t/2}|^2 = |1 + j|^2 = 2 \). Thus, \( x(t) \) is an infinite-energy signal while \( y(t) \) and \( z(t) \) are finite-energy signals. The power of \( y(t) \) and \( z(t) \) are zero because they have finite energy. The power of \( x(t) \) can be calculated by using the symmetry of the signal squared and letting \( T = NT_0 \):
Solution (cont.)

\[ P_x = \lim_{T \to \infty} \frac{2}{2T} \int_0^T \cos^2(\pi t/2 + \pi/4) \, dt = \lim_{N \to \infty} \frac{1}{NT_0} \int_0^{NT_0} \cos^2(\pi t/2 + \pi/4) \, dt \]

\[ = \lim_{N \to \infty} \frac{1}{NT_0} \left[ N \int_0^{T_0} \cos^2(\pi t/2 + \pi/4) \, dt \right] = \frac{1}{T_0} \int_0^{T_0} \cos^2(\pi t/2 + \pi/4) \, dt \]

Using the trigonometric identity

\[ \cos^2(\pi t/2 + \pi/4) = \frac{1}{2} [\cos(\pi t + \pi/2) + 1] \]
we have that

\[ P_x = \frac{1}{8} \int_0^4 \cos(\pi t + \pi/2)dt + \frac{1}{8} \int_0^4 dt = 0 + 0.5 = 0.5 \]

The first integral is the area of the sinusoid over two of its periods, thus zero. So we have that \( x(t) \) is a finite-power but infinite-energy signal, while \( y(t) \) and \( z(t) \) are finite-power and finite-energy signals.
Example

Consider an aperiodic signal \( x(t) = e^{-at}, \ a > 0, \) for \( t \geq 0 \) and zero otherwise. Find the energy and the power of this signal and determine whether the signal is finite energy, finite power, or both.
Consider an aperiodic signal $x(t) = e^{-at}$, $a > 0$, for $t \geq 0$ and zero otherwise. Find the energy and the power of this signal and determine whether the signal is finite energy, finite power, or both.

The energy of $x(t)$ is given by

$$E_x = \int_{0}^{\infty} e^{-2at} dt = \frac{1}{2a} < \infty$$

for any value of $a > 0$. The power of $x(t)$ is then zero. Thus, $x(t)$ is a finite-energy and finite-power signal.